Jonathan Gingerich

Valid and Invalid Arguments

An important part of philosophy is the study of arguments. An argument consists of a series of propositions, one or more of which are premises and one of which is a conclusion. The premise or premises of an argument provide evidence or support for the conclusion.

Here is an argument that is similar to an argument that Descartes famously advanced:

1. I think.
2. If I think, I exist.
3. Therefore, I exist.

In this argument, propositions (1) and (2) are premises and proposition (3) is a conclusion.

An argument is valid iff it is impossible for the premises of the argument to be true while the conclusion is false. Otherwise, an argument is invalid.

An argument is sound iff it is valid and its premises are true. Otherwise, an argument is unsound.

Valid Arguments

Here are two common types of valid argument:

Modus Ponens (short for modus ponendo ponens, or “the way of affirming by affirming”)

Consider the argument:
1. If it’s raining, then it must be cloudy.
2. It’s raining.
3. ∴ It must be cloudy. (∴ means “therefore.”)

If it’s true that it’s raining and that it if it’s raining then it must be cloudy, it has to be true that it’s cloudy. It’s impossible for both of the premises to be true but for the conclusion to be false, so the argument is valid. Hopefully, this seems very basic and intuitive. Here’s another example:

1. If Tim is in Paris then Tim is in France.
2. Tim is in Paris.
3. ∴ Tim is in France.

We can put this type of argument into symbolic form:

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* “Iff” stands for “if and only if”; it is an abbreviation commonly used by philosophers.
Arguments of this form are valid.

There are a few different ways that we can say $A \rightarrow B$:

- If $A$ then $B$ (or: If $A$ is true then $B$ is true)
- $A$ only if $B$
- $B$ if $A$
- If $A$ is true then $B$ is true
- $\neg B \rightarrow \neg A$ ($\neg$ means “not”)
- $\neg (A \land \neg B)$ ($\land$ means “and”)

We can also think about $A \rightarrow B$ graphically:

If something is in $A$, then it is also in $B$.

Modus Tollens (short for modus tollendo tollens, or “the way of denying by denying”)

Consider the argument:

1. If bats are birds then they have feathers.
2. Bats don’t have feathers.
3. ∴ Bats are not birds.

Like the examples of modus ponens, this argument is valid because its premises can’t be true while the conclusion is false, although it might not be as obvious. But suppose that the premises were true while the conclusion was false. In that case, we could make the following argument:

4. Bats are birds (which we get by supposing the conclusion to be false).
5. ∴. Bats have feathers (which we get from (1) and (4) by modus ponens)

But now we have a conclusion that contradicts premise (2). So we can’t make the conclusion, (3), false while the premises, (1) and (2), are true.

We can put this type of argument into symbolic form as follows.

1. $A \rightarrow B$
2. $\neg B$
3. $\neg A$
Arguments of this form are valid.

Think back to the diagram that we drew to illustrate A→B: if something isn’t in B, then it can’t be in A either.

Invalid Arguments

Here are three common types of invalid argument:

Affirming the Consequent

Consider this argument:

(1) If I’m sweating, I must be working out hard enough.
(2) I’m working out hard enough.
(3) ∴ I’m sweating.

Is this a valid argument? No, because both premises can be true while the conclusion is false. This might be easier to see with this argument:

(1) If it’s Saturday then it must be the weekend.
(2) It’s the weekend.
(3) ∴ It’s Saturday.

Of course, if it’s Sunday then (1) and (2) are true but (3) is false.

Symbolically, affirming the consequent looks like this:

(1) A→B
(2) B
(3) A

Arguments of this form are invalid. Keep in mind that it doesn’t follow from an argument being invalid that the conclusion of the argument is false—invalidity just means that it is possible for all of the argument’s premises to be true while its conclusion is false.

Denying the Antecedent

Consider the argument:

(1) If she is the author of To the Lighthouse then she is the author of Mrs. Dalloway.
(2) She isn’t the author of To the Lighthouse.
(3) ∴ She isn’t the author of Mrs. Dalloway.
Again, this argument isn’t valid because both premises can be true while the conclusion is false. Consider this argument.

(1) If I am a student in the course, then I can post on the Discussion Board.
(2) I’m not a student in the course.
(3) ∴ I can’t post on the Discussion Board.

But if I’m the instructor, then (1) and (2) are true while (3) is false.

Symbolically, denying the antecedent looks like this:

(1) A → B
(2) ¬A
(3) ¬B

Arguments of this form are invalid.

Missing Premises

Often, assumptions are left unstated because they are obvious or are implied by the context. For instance, “Sarah is only twelve years old, so she can’t vote” leaves unstated the premise that twelve year olds can’t vote. But often, missing premises aren’t so obvious. Consider the argument:

(1) If I can sell enough books, I’ll get rich.
(2) ∴ I’ll get rich.

As stated, this argument is invalid: it is possible for (1) to be true while (2) is false if I’m not able to sell enough books. This argument could be improved with the addition of an additional premise:

(1) If I can sell enough books, I’ll get rich.
(2) I can sell enough books.
(3) ∴ I’ll get rich.

Now, the argument is valid.