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Reflection Paper 2

Decision-making as Game: A Mode of Prediction and Solution

In a preschooler's game of make-believe the able observer will find many of the higher-order thinking skills that educators wish to teach later on in formal curricula. For example, in a game of School, participants assume the role of classroom characters—perhaps a teacher, sometimes a student, maybe a parent, or school custodian. Once roles have been assigned, players act out according to what each of their roles requires of them.

Here there are no codified instructions of play—yet even very young children are very keen to discover cheaters, i.e., players trying to execute inappropriate or illegal moves of play. Players in the student position cannot, typically, run the classroom, in the same way a player acting as a teacher can. Lacking a well-defined set of rules, players must rely on an internal assessment of the structure of classroom-life. The players must then respond to each other given the constraints of the rules of play, even though each player has devised her own set of rules (perhaps) independently from the rest. Play is dynamic. Consequently, each player must evaluate the situation, make predictive judgments and enact them according to the constraints of the game. Such analysis and subsequent action epitomize critical thinking skills. For these reasons, I believe that it is natural and useful to develop a philosophy of decision-making centered around the principles of gaming.

Before we go too far, it is instructive to say exactly what is meant by a game. Naïvely, an *n-player game* $G = [\![G^1 \mid \cdots \mid G^n]\!]$ is a collection of sets G^k of accepted moves[1], one set for each player k. It is important to note that the move sets G^k can be identical, as in Tic-tac-toe or chess, but in other games they need not be. In soccer, for example, the goalie has privileges not enjoyed by other players on the field. I choose to construe game as widely as is useful. In some situations, it may well be instructive to extend a game of soccer to include the referees, who themselves have separate move sets. For the game of School to be played well, one might expect the player students to act differently than the school nurse. Even then, the students may have different acceptable moves among themselves.

In every multi-player game, we have three parts: the *players* (or constituents) who engage in the game play, the *set of rules* (or culture) that directs how each participant is to act during the game, and the location of the game, called the game's *court*. The first two components of a game render it, by definition, a community. As such, all the previous discussion of community applies to gaming. It makes

[1] In combinatorial game theory, a game is defined inductively, beginning with the *endgame* $\{\,|\,\}$. Then if if G^L and G^R are both games, then $G = \{G^L \mid G^R\}$ is also a game. In this definition there are only two players, Left and Right. In the first move, Left chooses a move $H = \{H^L | H^R\}$ from G^L and then turns the board over to Right. From Right's perspective, the board represents a brand new game H that is simpler than the original game G. Right chooses a move from H^R, which is itself another game, and politely presents it to Left. In standard play, the game continues until one player cannot move. The last player to move wins. In our analysis, we cannot hope our games will be nearly so structured. To wit, in many social situations, the game grows increasingly complicated with each move—not less so.

sense that strategies employed for good gaming might well find their way in the context of communities. Indeed, the requisite parts of a community are identical to the components of a multi-player game, the only defining differences are nothing more than ghosts of the language used to describe either structure. I believe we can leverage this insight to create more effective learning environments.

For each participant in a game there is at least one objective. Throughout the duration of a single game, a player may host several goals of varying immediacy. In a game of squash, the player who serves may act aggressively with the aim of scoring a point, but may later act defensively when her opponent wins the serve, for instance. In a game of School, it is difficult to determine what each player's objective is at any given time. Indeed, most games found in society are of this nature. Players operate on incomplete sets of knowledge about the current state of the game. In a real-life game of telephone, one player calls the other using a telephone with the objective of talking with the other player. However, the player initiating the call has no idea whether the other player is near the telephone, wants to talk, or is even awake. Despite these trifling though practical concerns, many people manage to play and succeed at playing telephone everyday. Few players, however, realize that they are playing a game. Only in extended play do some finally name the game rightfully as "phone tag."

In so many games (be they taken from real-life or toy chest), players who can predict other participants moves well have an advantage. The benefits of accurate predictions do not arise *per se*, but from their ability to enable smart decision-making. If I know what you are most likely to do next, I can plan my own moves accordingly—as in a game of squash.

The professional academic researching community predicates its practices on the principles of gaming described above. All fields have common to them pattern-searching. In fields like marketing, navigation, and politics, pattern-driven predictions are of public, cardinal importance. These patterns, when considered in their appropriate realms of persistence, serve as a model to predict presently unknown behaviors or other internal structures. Rule sets are nothing more than patterns which persist over a period of time that happen to be beneficial to participants for their attaining objectives in play. When a pattern has spent its time of usefulness, it is abandoned; the rules change, though the game may play on. Paradigm shifts in science illustrate this point nicely. However, examples need not be so drastic. In history, researchers piece together details of information describing motive, social climate, geographical constraints, and the like in order to predict their consequences as a whole. Because historical variables are apt to change over time, patterns that dominate analysis of the American Revolution may be of little value to an analysis of the game of the Harlem Renaissance. The scope of a pattern is necessarily limited. Therefore not all rules apply to all players at all times.

Historians must take into account a diverse range of information in order to make effective (predictive) judgments. So, too, good gamers will respond to subtle cues encoded in the actions of other players and the game's court. They know that all actions communicate information. As such, they can glean information that other participants do not explicit offer. Because game play hinges as much on the specific players involved and the particularities of the court as the rules themselves, games are not a symbolism in the full sense of the term. Here it is helpful to distinguish between *the* game and *a* game.[2]

In *the* game of tennis, there are certain rules that each player must adhere to. The details of the players themselves are irrelevant to the rules of the game. Impartial referees of games base their calls in the semantics of the game. For them, all the moves in tennis take place in a *semantic field.* Meanwhile, the specific strengths, weaknesses, and over-all strategies of each player will differ from one another. If you play me at a game of tennis, I advise you play to my backhand, because my backhand is somewhat weak. However, I know how the ball will respond to a clay court well. If you have little experience playing tennis on a clay court, then I have an advantage. Details such as these are steeped in the physical reality of the game, what I like to call the *physical field* of game play. On the physical field, the representation of the game effects play. Here we cannot freely substitute one player for another without charge (as is the case on the semantic field). A player X differs from player Y in important ways on the physical field. (Would you rather a set of tennis with me or the current Wimbledon champion?)

Particular instances of a game operate differently from the game's generic form. From now on, I will call *a* game (of tennis, for example) a *match.* Most games are played in matches. Few are played soley in some abstract, semantic field.[3] Matches combine elements from both the semantic and physical fields of the game. Because details born in the physical field cannot be freely interchanged (as is the case in the semantic field), game play takes place in a location that is hard to describe. To use previous papers as a model, I suggest calling the true location of game play a *game space.*

Game spaces are useful, if you recognize that you're in one. Knowing an order exists—even if it is intractably complicated—often can yield practical, realizable benefits. While the complete nature of the set of rules may remain unknown, partial descriptions constrain the form a solution (winning strategy) must take. Thus, using game-theoretic framework for decision-making is helpful in any situation, not only in the highly stylized form of situations traditionally thought of as games. Framing any interaction as a collection of moves in a game space does not necessary offer a new perspective on the situation. Instead, it allows players to see more fully their own perspective and the associated consequences. To wit, games set up an environment which naturally encourages meta-cognitive processes. Players must ask themselves whether their intended action is in accord with proper game play, and also, whether such moves will advance their positions. Game play requires a heightened awareness of its players. To become a better player, one must develop a deeper understanding of the rules. In turn, because of the structure presented by a game space, players can glean more information from their actions than if the set

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[2]Readers familiar with the mathematical notion of an equivalence relationship can think of a game as an equivalence class of individual games, where equality of sets of rules identify two members of the class. For those readers with an object-oriented programming background, each abstract game forms a class, individual instances (matches) of the game can be thought of as objects within that class.

[3]A systems approach, say, to mathematics, which incorporates societal forces shows that even academic disciplines which are normally thought to lie entirely in the semantic field are, in reality, also played in matches. Professional mathematical journals still *referee* their articles. Even here the language implicates the underlying ludic structure.

of rules were ignored entirely. Knowing that there are rules is, itself, a useful bit of knowledge.

Here I would like to work through an artificial example taken from the world of consulting interviews (which is a game all its own) to show the power of a games perspective to problem-solving. Please excuse its lengthy set-up.

> Elizabeth, Brian, Dean, and Leslie want to cross a bridge. They all begin on the same side and have only 17 minutes to get everyone across to the other side.
>
> To complicate matters, it is night and there is only one flashlight. A maximum of two people can cross at one time. Any party that crosses, either 1 or 2 people, must have the flashlight with them. The flashlight must be walked back and forth, it cannot be thrown.
>
> Each student walks at a different speed. A pair must walk together at the rate of the slower student's pace.
>
> | | |
> |---|---|
> | Elizabeth: 1 minute to cross. | Brian: 2 minutes to cross. |
> | Dean: 5 minutes to cross. | Leslie: 10 minutes to cross. |
>
> For example, if Elizabeth and Leslie walk across first, 10 minutes have elapsed when they get to the other side of the bridge. If Leslie returns across the bridge with the flashlight, a total of 20 minutes has passed, and you have failed the mission.

I suggest you try to work out a solution on your own before you read on. After all, I've presented you with a game and a chance to play.

Unlike in a game of School, whose rule set is largely unknown, there are only a handful of acceptable moves. Each is of the form "two kids go," which I'll denote symbolically by $(\rightarrow\rightarrow)$, and "one comes back" $(\leftarrow)$. I have presented the game deliberately to rule out solutions that involve Dean's swimming across or Leslie's impromptu disappearance, etc. All valid solutions must consist of the two types of moves described above. Because we know the exact form of each move, we can make some general observations about how to play. With these observations in mind, hopefully, we will arrive at a solution. Note that right now we do not know any thing about the solution of than its form. Namely,

$$(\rightarrow\rightarrow) \leftarrow (\rightarrow\rightarrow) \leftarrow (\rightarrow\rightarrow).$$

Observation 1 Firstly, not all times factor into the calculation. In the first kind of move $(\rightarrow\rightarrow)$, on the slower of the two times counts. To reflect this observation, I will let a longer arrow reflect the relative amount of time it takes players to cross the bridge. The faster traveler receives a short arrow; the slower one receives a longer one. Together, the move is $(\rightarrow\longrightarrow)$. From a computational point of view, this observation means that with each move of the first kind, the slower traveler "hides" the fact that the second one is crossing, too.

$$(\rightarrow\longrightarrow) \longleftarrow (\rightarrow\longrightarrow) \longleftarrow (\rightarrow\longrightarrow)$$

It is also useful to notice that Leslie can (and must) cross only once, otherwise we will already spend more than our allotted time with her moves alone. Likewise, Observation 2 not everyone can carry the flashlight. The combined times of all the travelers add up to 18 minutes, which is over our target. In any solution, at least one person has to be completely hidden by someone slower. Since we already know that Leslie cannot make a return trip, Observation 2 means that additionally either Elizabeth, Dean, or Brian can never make a return trip as well. Now we only need to pay attention to the long arrows.

We should also observe, that unlike Leslie, Elizabeth hides no one else's time Observation 3
while crossing. Because of this, Elizabeth cannot carry the flashlight back on both trips. If she did always make the return trips, she'd have to cross in the other direction in pairs three times. That means all four times would figure into the total time it takes to get across the bridge. And we know by Observation 1, that that's against the rules. To figure out which two students should make the return trips, we notice that the sum of Brian, Dean, and Leslie's times is 17 in only three moves. The form of a valid solution must be at least 5 moves long. The rules of the game force us to select Brian and Elizabeth on the return trips. In fact, this observation constrains the solution even more. Leslie and Dean must travel together. If they do not, they will each hide the times of their companion. However, that means that all four of the times will appear in the solution, and we know that in any valid solution, that's impossible. The constraints given by our observations so far may be written like this:[4]

$$(\to\longrightarrow)\ \underset{2}{\overset{\text{Brian}}{\longleftarrow}}\ (\to\longrightarrow)\ \underset{1}{\overset{\text{Elizabeth}}{\longleftarrow}}\ (\to\longrightarrow)$$

It's now time to make one final observation: a person must cross the bridge once Observation 4
before returning. We implicitly used this fact above in our analysis of Observation 3. Because Elizabeth and Brian come back, we need to decide how to get them across in the first place. In the solution we've decided to construct, either the long or short arrow in the first trip must be occupied by Brian because he returns in the second move. Since Leslie and Dean must travel together, neither can take the other spot. By process of elimination, we see that Brian and Elizabeth make the first crossing over the bridge in any solution.

$$\left(\overset{\text{Elizabeth}}{\to}\ \underset{2}{\overset{\text{Brian}}{\longrightarrow}}\right)\ \underset{2}{\overset{\text{Brian}}{\longleftarrow}}\ (\to\longrightarrow)\ \underset{1}{\overset{\text{Elizabeth}}{\longleftarrow}}\ (\to\longrightarrow)$$

As was mentioned previously, Leslie and Dean must travel together. So we have no other choice but to send them next. Brian and Elizabeth make the final crossing. In doing so, they have completed the challenge and we have derived a solution to the game merely by constraining the form it could take—by inferring information from the rules of the game.

$$\left(\overset{\text{Elizabeth}}{\to}\ \underset{2}{\overset{\text{Brian}}{\longrightarrow}}\right)\ \underset{2}{\overset{\text{Brian}}{\longleftarrow}}\ \left(\overset{\text{Dean}}{\to}\ \underset{10}{\overset{\text{Leslie}}{\longrightarrow}}\right)\ \underset{1}{\overset{\text{Elizabeth}}{\longleftarrow}}\ \left(\overset{\text{Elizabeth}}{\to}\ \underset{2}{\overset{\text{Brian}}{\longrightarrow}}\right)$$

In a very real sense, the problem and the solution of the game were both given to us at the same time. It merely took a careful reading of the rules of the game to determine a winning strategy. For most games, a complete solution cannot be found in the rules alone. Elements of chance and ignorance of the full set of acceptable moves stymie such simple analyzes. However, as this example shows, a game theoretic mindset can markedly simplify solution-finding by pruning away strategies which violate the rules of the game.

[4]There are other solutions. There is a symmetrical ambiguity in Observation 3. It does not tell us who should take which return trip. This symmetry is reflected in the final set of solutions.