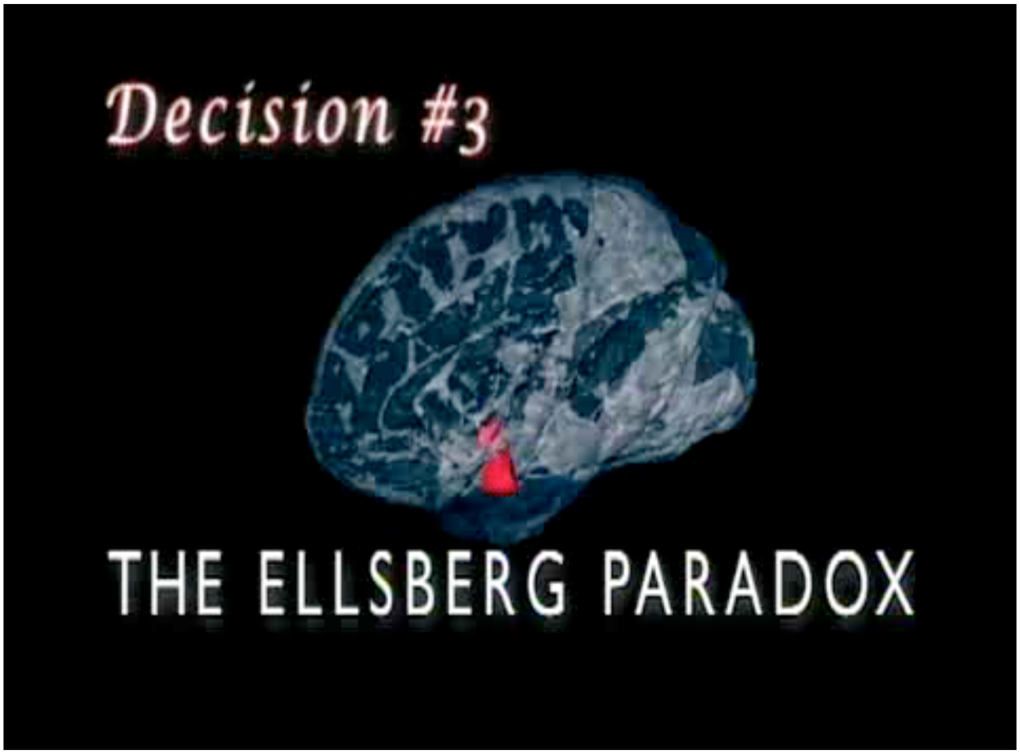

Friday, May 6, 2011
Daniel Ellsberg

What paradox?

I learned long ago to try to resist requests by laymen who have heard of an “Ellsberg paradox” to explain to them what it is. There’s no way to describe my argument that doesn’t lead to the query, “So, what’s the paradox?” Convincing them that they don’t always act as if they assigned precise numerical probabilities to uncertain events needs no demonstration for nearly anyone other than ordained Bayesian statisticians. Even showing that their choices among uncertain gambles can’t always reveal that they even regard one particular event as more likely/probable, less likely or equally likely compared to another—while this proposition is less “obvious”--seems hardly challenging or noteworthy. The puzzle for them is why these common-sense propositions are described as paradoxical.

Since I agree with them on this, I can only say, “You’re right, there is no Ellsberg ‘paradox.’ I’ve never used that term (except in quotes.)” But to explain why others have (if I’ve allowed this discussion to get started), I go on to explain that a very, very smart statistician, L.J. Savage, convinced himself sixty years ago and went on to convince several generations of smart students and followers that this common-sense understanding--that not all uncertainties can be expressed adequately by precise numerical probabilities or even by a complete ordering of relative likelihoods-- was unsound (like, say, the once common-sense belief that the earth is flat or that the sun goes around it).

They came to believe—following Savage’s brilliant argument in *The Foundations of Statistics* (New York, 1954)-- a contrary proposition. Namely, that all “reasonable” people would *want* to behave under all conditions of uncertainty-- if they reflected on their choices in the light of this new analysis—*as if* they assigned definite "utility" numbers to all possible outcomes of their alternatives and definite probabilities to the events or propositions bearing on these outcomes. In fact, these numbers could be actually be inferred from certain of their observed, reflective choices. (I don't try to expound to non-specialists—and obviously don't need to for the present readership— the next point about the unique rationality of choosing so as to maximize the mathematical expectation of utility: Daniel Bernoulli's proposition of 1738, resuscitated by these "neo-Bernoullians," in I.J. Good's phrase. Nor the logical derivation of the Bernoulli proposition from sets of "axioms," by Savage and others, that seem individually to be very persuasive normative rules for "rational" or reasonable choice.)

These "Bayesians," as they prefer to call themselves, were so sure of this prescription for rational behavior under uncertainty that when I brought to their attention, starting in 1958, hypothetical situations in which many otherwise-reasonable people, and even many of these decision-theorists themselves, did *not* want to choose, even on reflection, as their normative theory postulated, they found this paradoxical. From their point of view, it was if I was arguing with surprising plausibility that it could still be reasonable to believe that the earth *is* flat, after all, and the sun revolves around it.

But wait a minute: to the uninitiated, that metaphor looks turned around. Weren't *they* the ones who were wrong: the ones who thought they knew otherwise, how all reasonable people ought to want to choose in the presence of uncertainty? Well, yes, that's what I think. I see the people who persisted in the belief that the Savage axioms were or should be universal norms—even if they weren't descriptive—as being the flat-earthers. So it goes; today's common sense may be yesterday's paradox, and vice versa.

As Savage reminded me in conversation, he had himself noted earlier that the pre-Bernoulli belief in the unique rationality of maximizing the mathematical expectation of *money* "was at first so categorically accepted that it seemed *paradoxical* to mathematicians of the early eighteenth century that presumably prudent individuals reject the principle in certain real and hypothetical situations." (*Foundations,* p. 92, emphasis added. Bernoulli appended to his paper a famous thought-experiment in which virtually everyone rejected the earlier principle of maximizing the mathematical expectation of money; it was known as the "St. Petersberg Paradox.") What I came to believe and sought to demonstrate between 1957 and 1960 was that the Bernoulli/Savage (Ramsay/de Finetti/ Luce and Raiffa, Rubin, et al) prescription of maximizing the mathematical expectation of *utility*—derived from their respective axiom-sets-- was just as inadequate, misleading and wrongheaded as a universal normative (or descriptive) principle as the pre-Bernoulli one of maximizing the mathematical expectation of money.

Origins

I came to this position and to the counter-examples that bolstered it neither out of the blue, nor from my undergraduate work—my Harvard honors thesis on "Theories of Rational Choice Under Uncertainty: the Contributions of von Neuman and Morgenstern—nor from discussions with RAND consultants like Savage or Tom Schelling (my mentor on bargaining theory, later formally my Ph.D. thesis advisor) or anyone else. The now well-known three-color urn experiment and the two-urn experiment (which came to me first) were the result of endless trial and error with paper and pen.

I was searching for choices between gambles—actions with uncertain outcomes—that would give operational meaning behaviorally for the first time to Frank Knight's distinction between "risk"—roulette-like gambles with "known," precise probabilities—and "uncertainty," when no such probabilities were "known." Savage's axioms implied that for the reflective choices of reasonable people, no such distinction could be inferred from their behavior; whatever their subjective feelings (about which he didn't speculate) they would act as if all uncertainties were "risks," whose numerical probabilities could even be measured by observing their choices. ...

What I was looking for, and eventually found, were choices among gambles that would unequivocally show a behavioral effect of differences in information and subjective confidence: differences that would show up in systematic and deliberate violation of one or more of the axioms by some persons that Savage and his followers would recognize as otherwise reasonable.

I intuited that these might be found in situations of high "ambiguity": "where available information is scanty or obviously unreliable or highly conflicting; or where expressed expectations of different individuals differ widely; or where expressed [or felt] confidence in estimates tends to be low…[like] the results of Research and Development, or the performance of a new president, or the tactics of an unfamiliar opponent…" ("Risk, Ambiguity and the Savage Axioms" *Quarterly Journal of Economics*, 1961)