# THE ULAM SPIRAL AS A GAME OF LIFE CELLULAR AUTOMATON

KARTHIK NARAYANASWAMI

*ABSTRACT: We use a distribution of prime numbers within the Ulam Spiral as the starting point for an instance of Conway's Game of Life cellular automaton. We summarize the characteristics of the resultant pattern.*

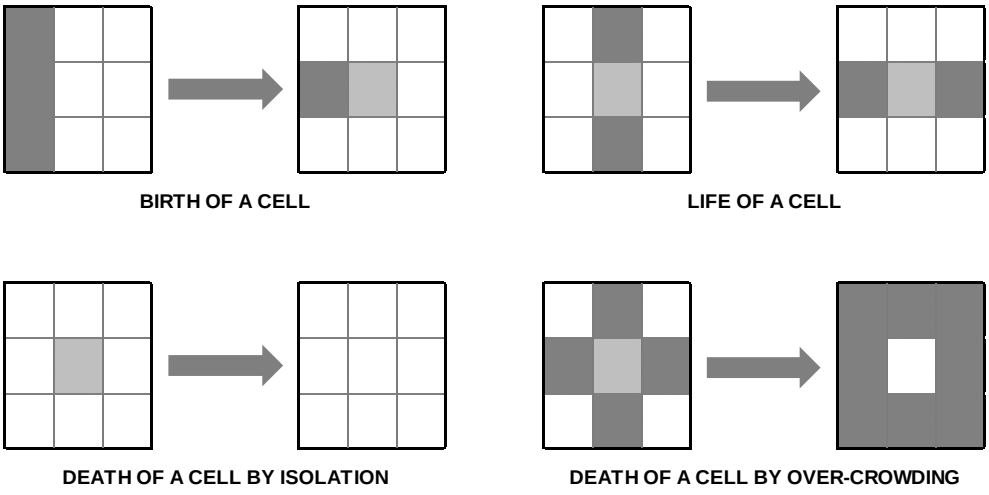
## 1. INTRODUCTION

The Ulam Spiral is a rectangular grid created by Stanislaw Ulam in 1963 to help visualize prime numbers (Ulam, 1964; 1967). It demonstrates the tendency of some polynomials to generate unusually large number of primes and the tendency for prime numbers to line up along diagonal lines. This occurrence of prime numbers along diagonals can be seen even in large spirals as well as in those spirals without 1 as the starting point.

The Game of Life is a zero-player game created by John Conway in 1970. It is a simplified cellular automaton whose outcome is entirely dependent on the initial state. Played in a grid, the rules of the Game of Life are as follows, for each iteration of the automata (Gardner, 1970):

1. Any dead cell with exactly three live neighbors becomes a live cell (*Birth*)
2. Any living cell with exactly two or three living neighbors continues on to the next generation (*Living*)
3. Any lone living cell with no living neighbors will die (*Death by Isolation*)
4. Any living cell with more than three living neighbors dies (*Death by Overcrowding*)

Based on these rules, a grid may settle on a stable state – however, even in its stable state, certain automatons would alternate between the two birth positions, which we will observe below.



## 2. RESULT

For our simulation, we limit the size of our square grid to 100x100, i.e. the numbers 1 – 10,000. Details, including numbers, can be seen by zooming into the grid.

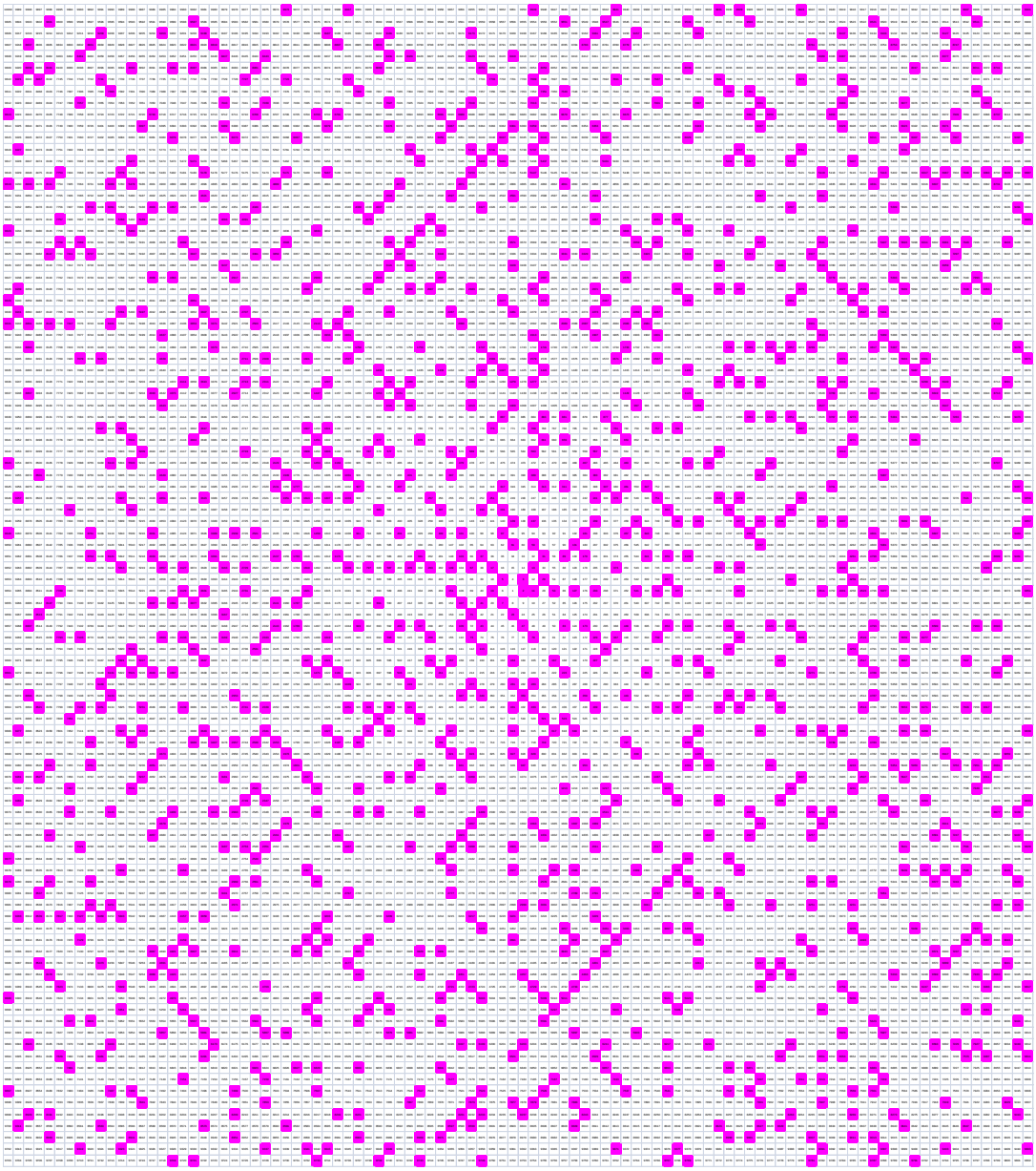


Fig. 1: Ulam Spiral of 1-1000 showing the distribution of Prime Numbers

We observe below one of the stable states of the automaton, after 1348 iterations.

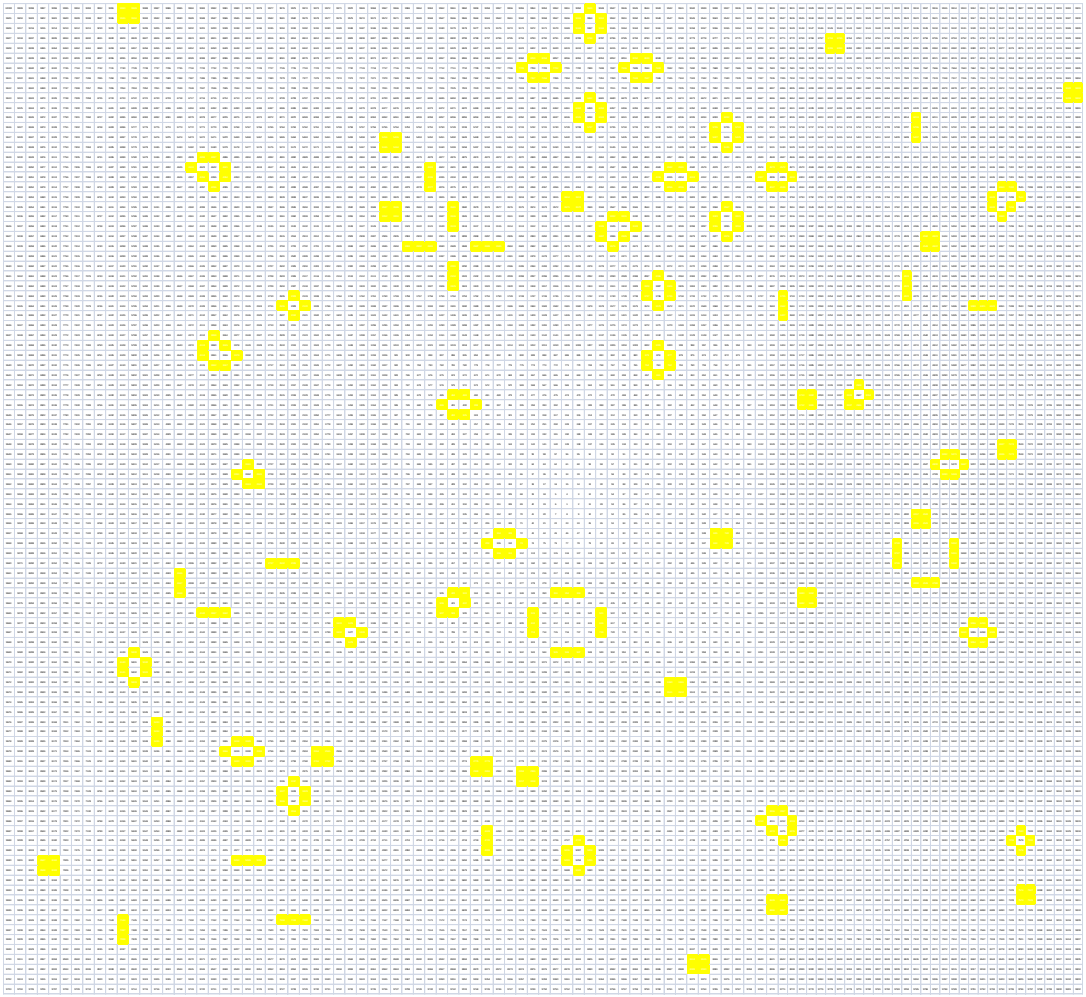


Fig. 2. Stable State (I) of Game of Life based on the Ulam Spiral

After 1349 iterations, we observe the following alternate stable state:

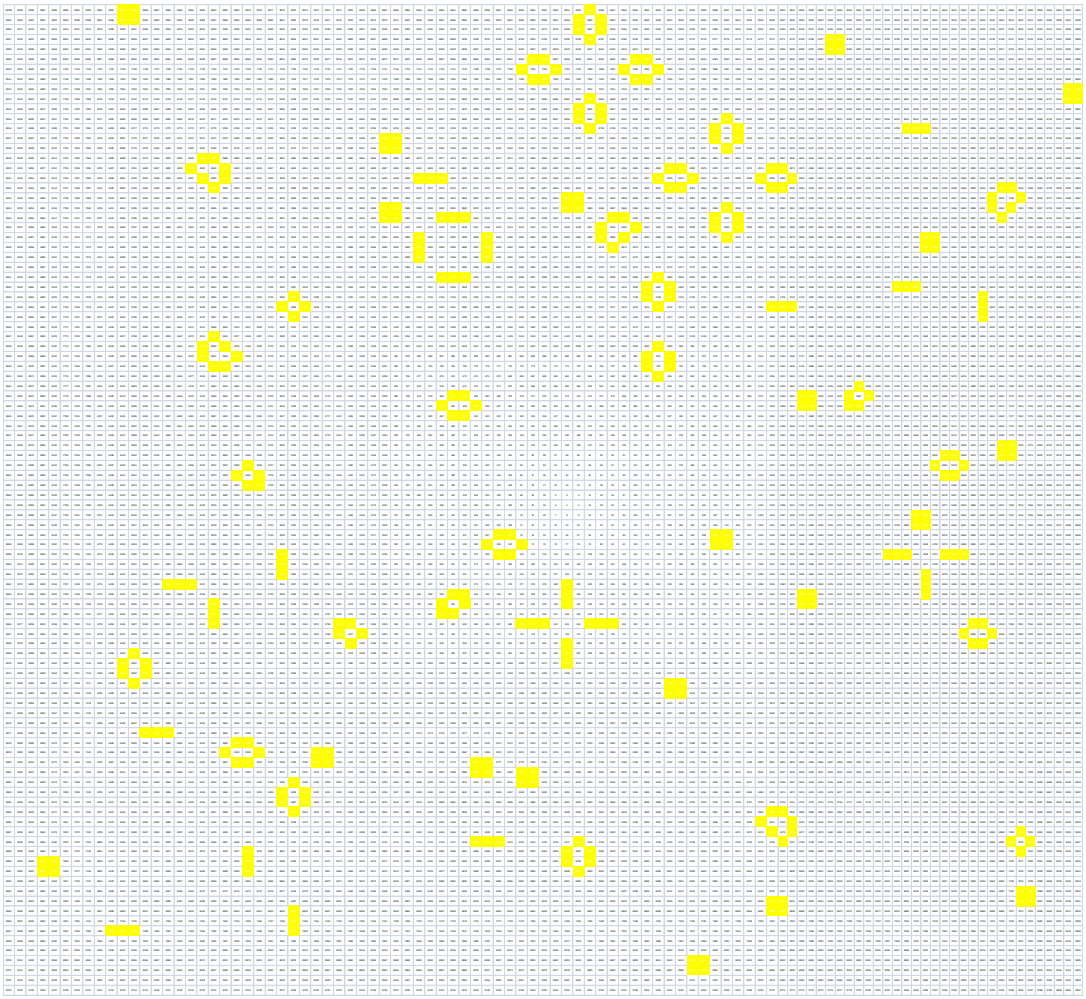


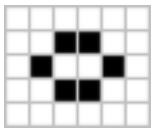
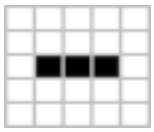
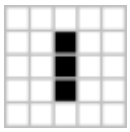
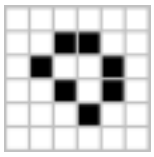
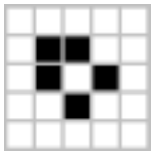
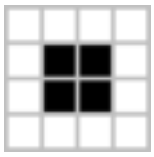
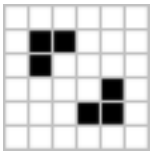
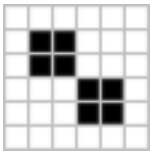
Fig. 3. Stable State (II) of Game of Life based on the Ulam Spiral

## 3. SUMMARY OF RESULTS

A summary of simulation results are given below:

| | |
|---|---|
| Total number of iterations | 1348 for Stable State (I)<br>1349 for Stable State (II) |
| Total number of stable "lives" | 73 |
| Total number of overlapping primes<br>(i.e., between initial and final stable states) | 42 in Stable State (I)<br>39 in Stable State (II) |
| Total primes in simulation | 1230 |

Furthermore, a summary of the Game of Life patterns are also given below:

| | |
|---|---|
| Total beehives  | 18 |
| Total blinkers   | 24 |
| Total loafs  | 5 |
| Total boats  | 3 |
| Total blocks  | 20 |
| Total beacon   | 1 |

## 4. REFERENCES


1. Stein, M.; Ulam, S. M. (1967), "An Observation on the Distribution of Primes", American Mathematical Monthly (Mathematical Association of America) 74 (1): 43–44.
2. Stein, M. L.; Ulam, S. M.; Wells, M. B. (1964), "A Visual Display of Some Properties of the Distribution of Primes", American Mathematical Monthly (Mathematical Association of America) 71 (5): 516–520.
3. Gardner, Martin (1970-10). Mathematical Games - The fantastic combinations of John Conway's new solitaire game "life". 223. pp. 120–123. ISBN 0-89454-001-7.